

# Propulsion

## Propeller-Hydrodynamics



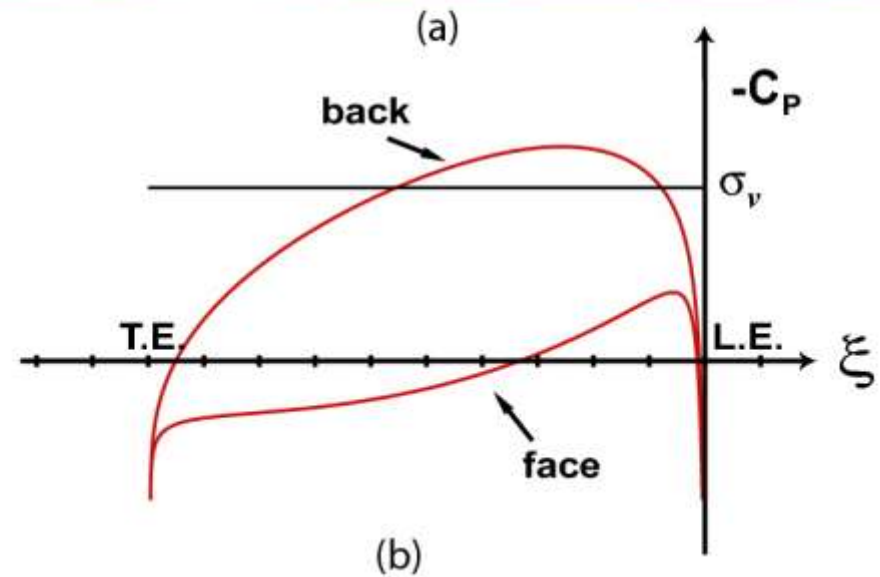
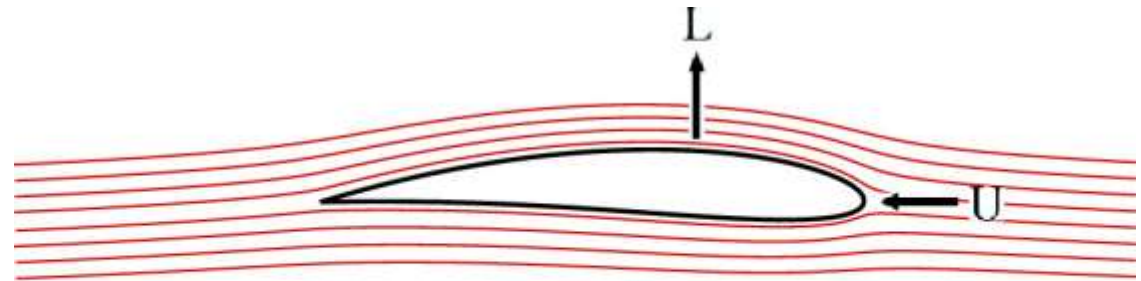
## 2 Force acting on 2-D foil



### ❖ Bernoulli 방정식

$$p + \frac{1}{2}\rho V^2 = p_\infty + \frac{1}{2}\rho U^2$$

$$C_p = \frac{p - p_\infty}{0.5\rho U^2}$$



# Lifting force represented by circulation

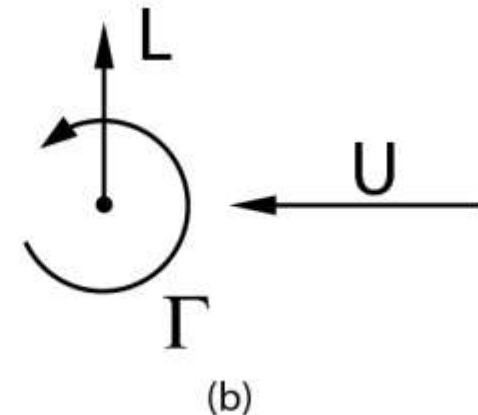
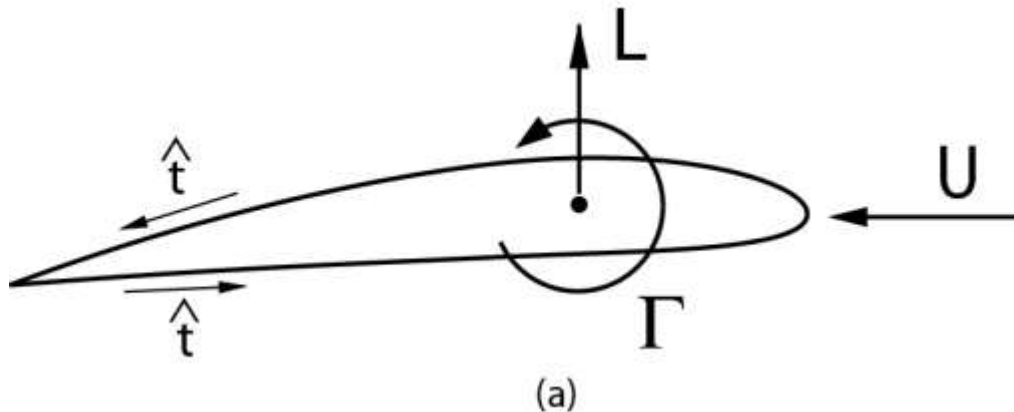


❖ Circulation:

$$\Gamma = \int \vec{V} \cdot d\vec{s}$$

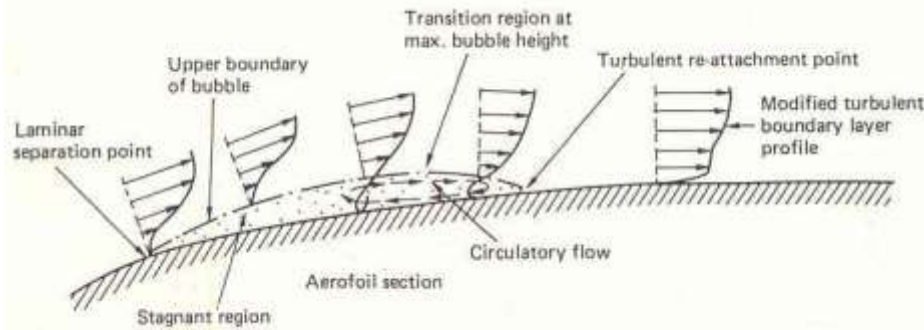
❖ Kutta-Joukowski formula:

$$L = \rho U \Gamma$$

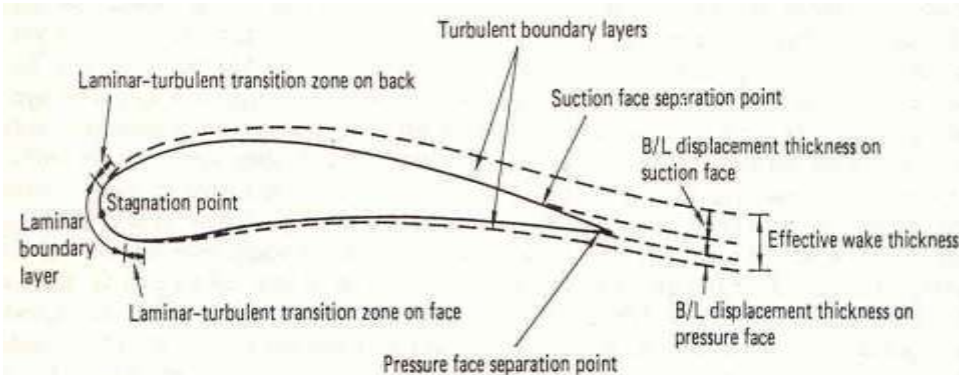


**Point vortex representing the circulation around a foil**

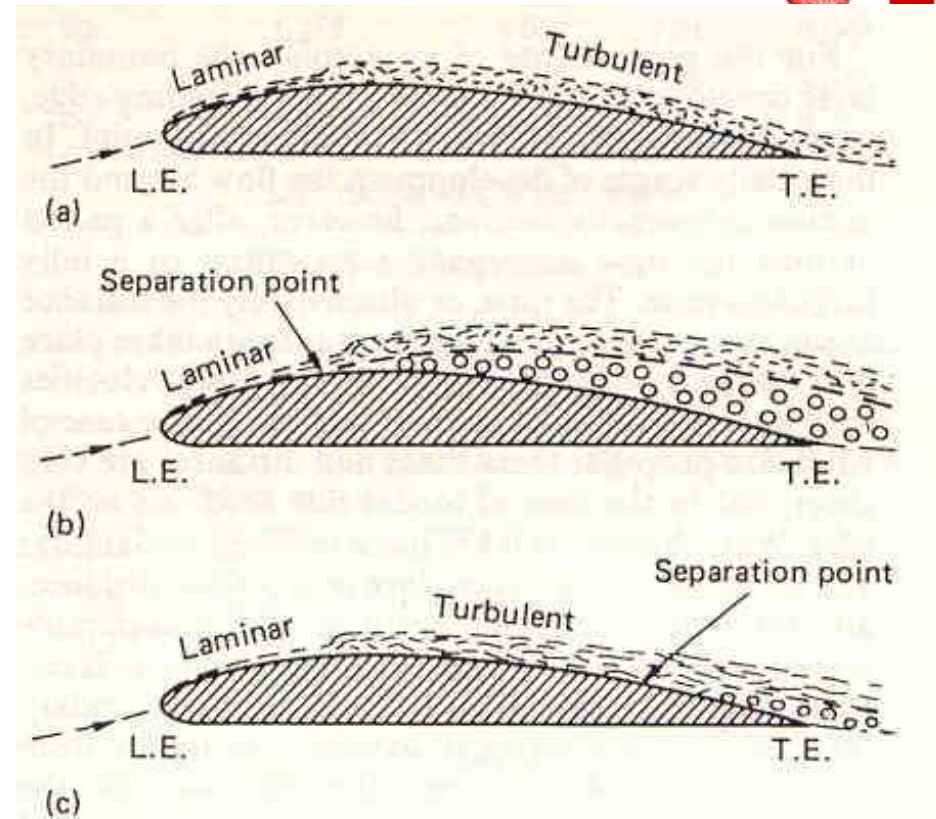
# Boundary layer growth(viscous effect)



Laminar separation bubble

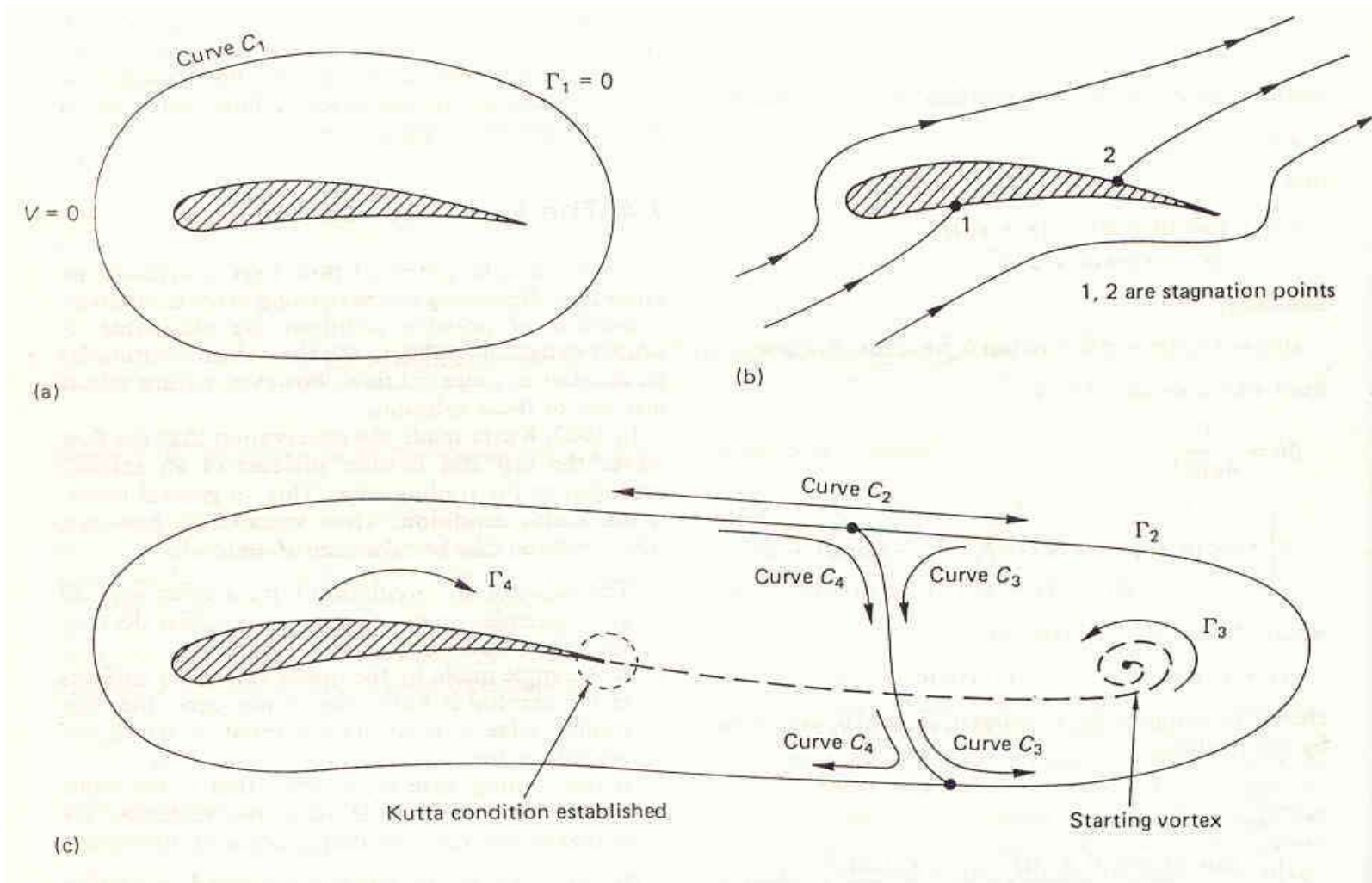


Boundary layer structure



Schematic flow regimes over the suction surface of an aerofoil : (a) fully attached laminar followed by turbulent boundary layer flow over suction surface; (b) laminar, leading edge separation without reattachment of flow over suction surface; (c) laminar followed by turbulent boundary layer with separation near the trailing edge

# Kutta condition

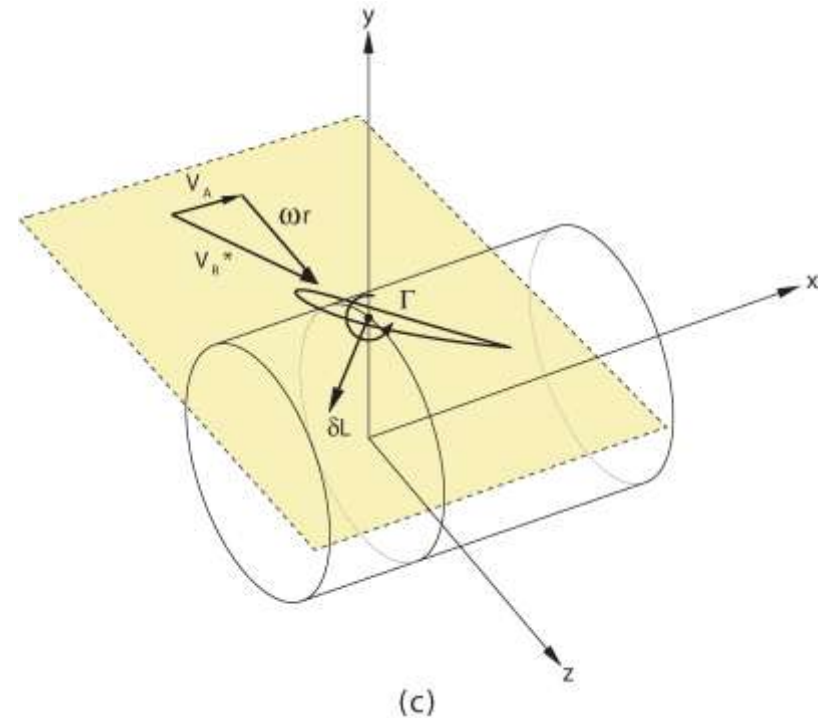
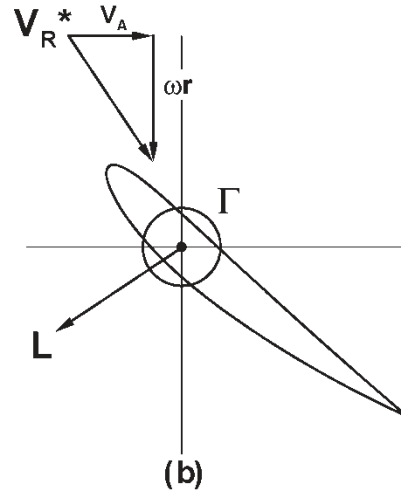
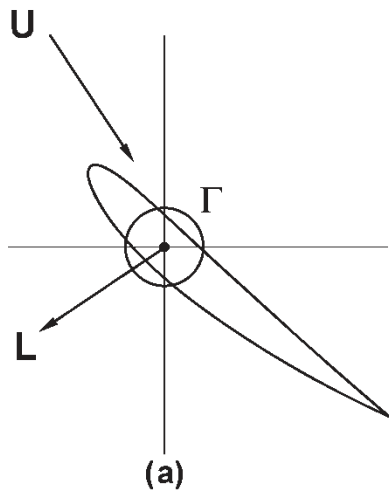


Establishment of the starting vortex : (a) aerofoil at rest; (b) streamlines on starting prior to Kutta condition being established ; (c) conditions at some time after starting

# Loaing on propeller section



❖ On-coming velocity:  $V_R^*(r) = \sqrt{(V_A(r))^2 + (\omega r)^2}$



Lift-generating mechanism on s 2-D blade sections (a, b) and its application to propeller blade section at a typical radius (c)

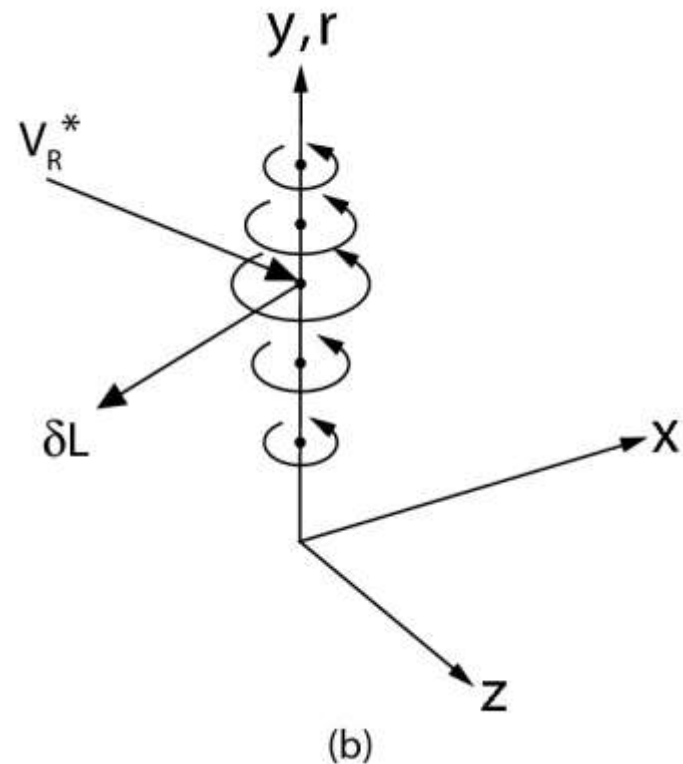
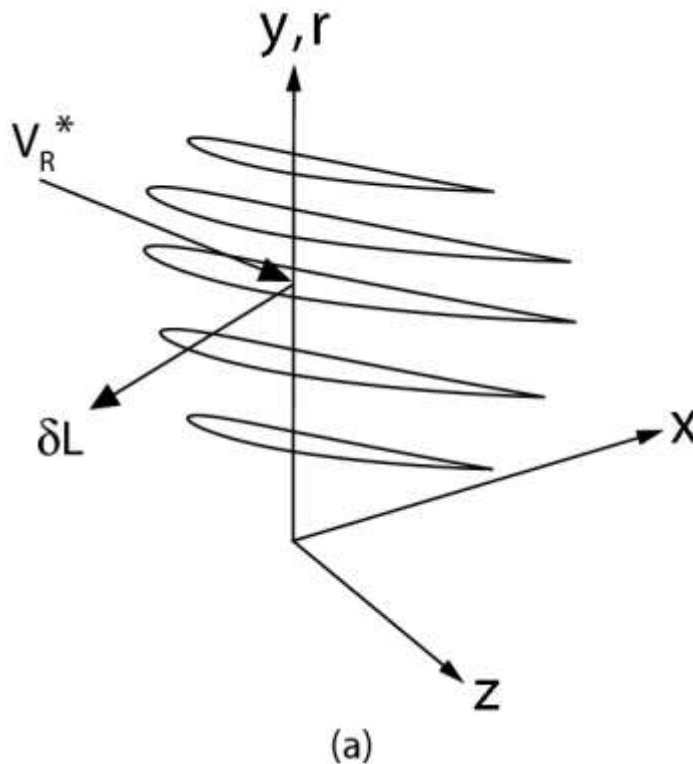


# Circulation distribution



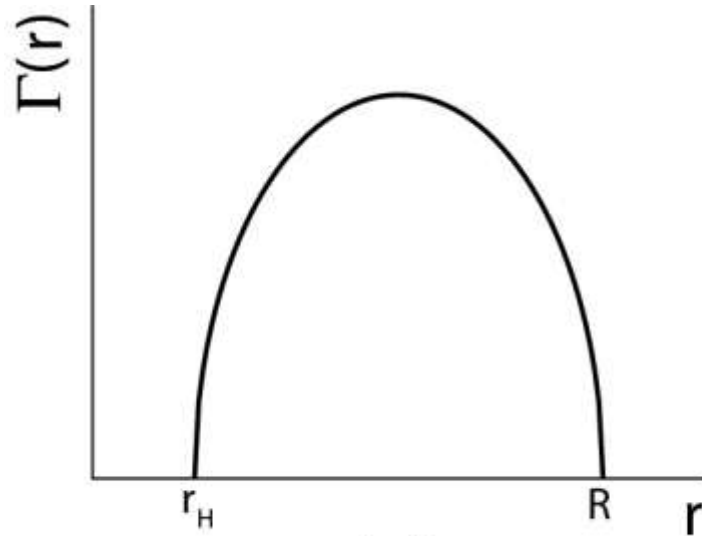
❖ Kutta-Joukowski formula:

$$\delta L(r) = \rho V_R^*(r) \Gamma(r)$$

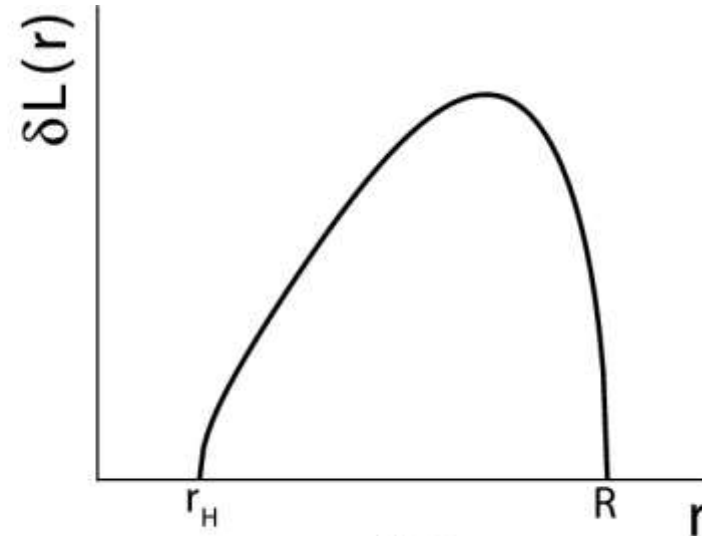


Building-up of propeller blade sections and lift(a) and  
Circulation replacing blade sections(b) at various radii

# Circulation and Load Distribution



(a)



(b)

Optimum: Elliptic distribution

$$\delta L(r) = \rho V_R^*(r) \Gamma(r)$$



# Propeller Velocity Diagram

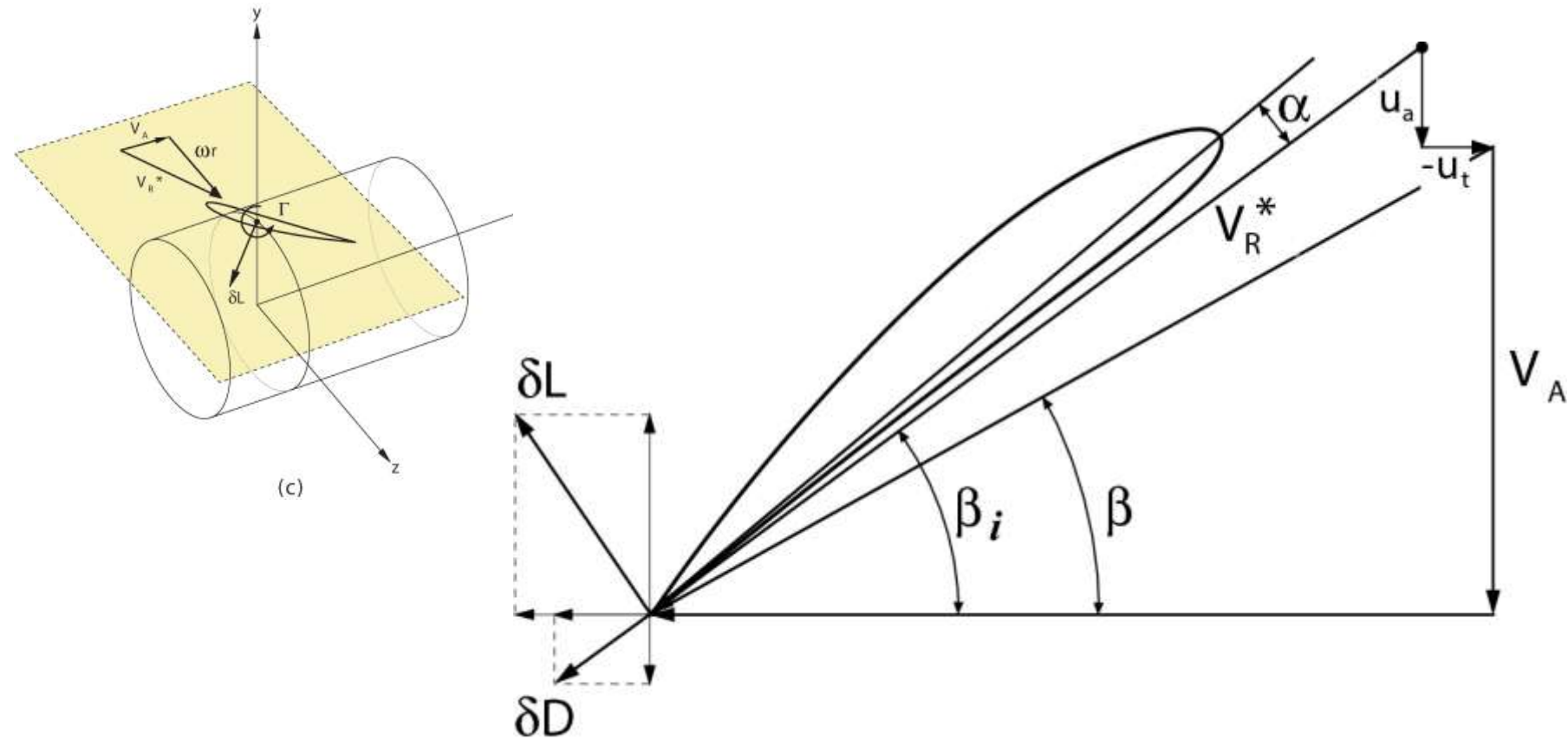


❖ Propeller induced velocity:

$$(u_a, u_r, u_t)$$

❖ Hydrodynamic pitch angle:

$$\beta_i = \tan^{-1} \frac{V_A + u_a}{\omega r + u_t}$$

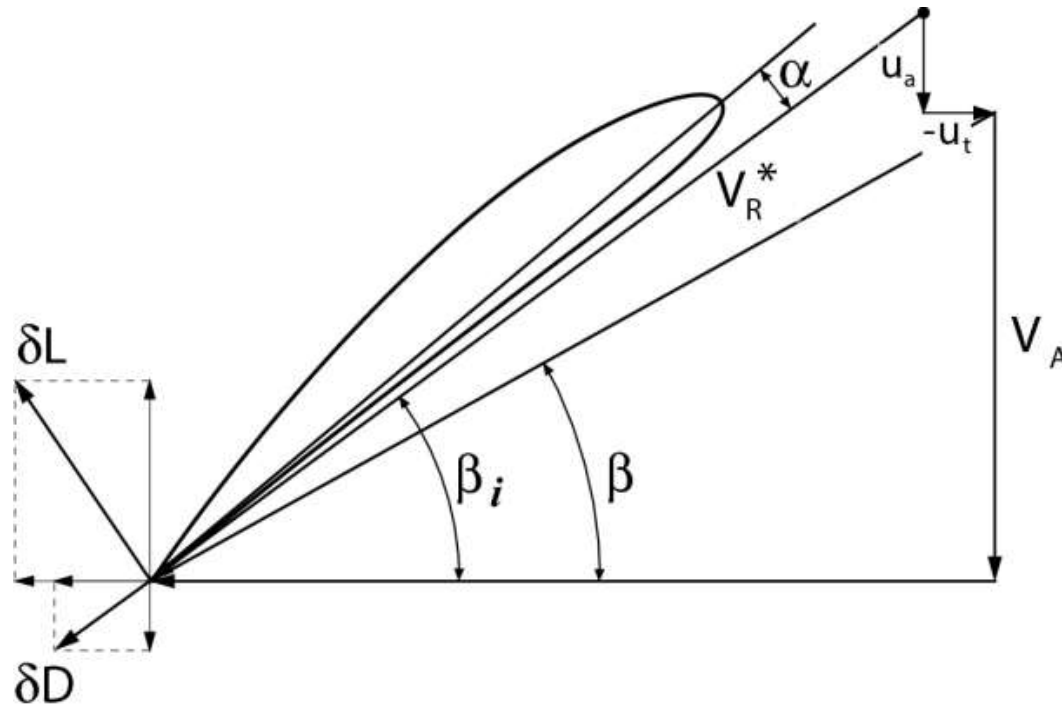


# Acting forces on propeller section



$$T = Z \int_{r_H}^R \delta T dr = Z \int_{r_H}^R (\delta L \cos \beta_i - \delta D \sin \beta_i) dr$$

$$Q = Z \int_{r_H}^R \delta Q dr = Z \int_{r_H}^R (\delta L \sin \beta_i + \delta D \cos \beta_i) r dr$$

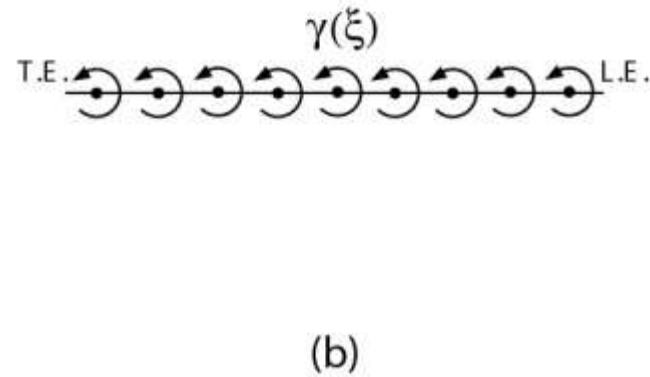
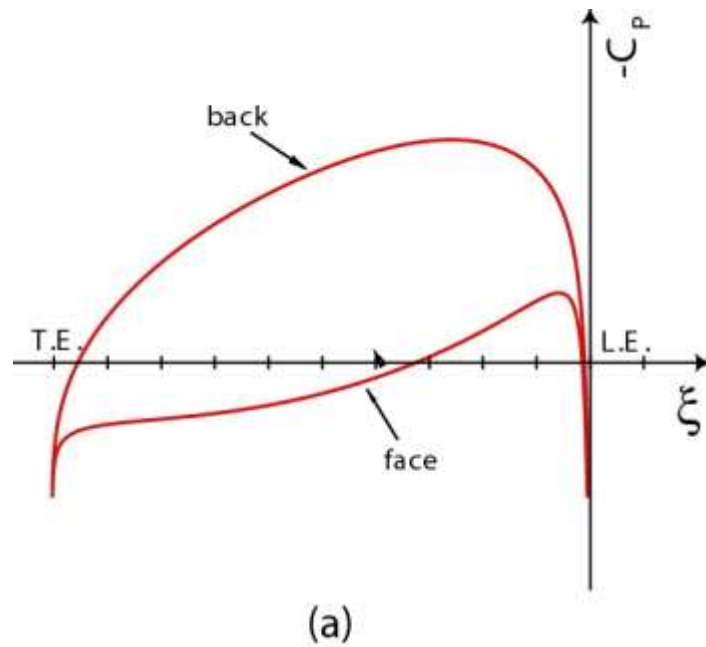


# The relation of pressure jump and vorticity

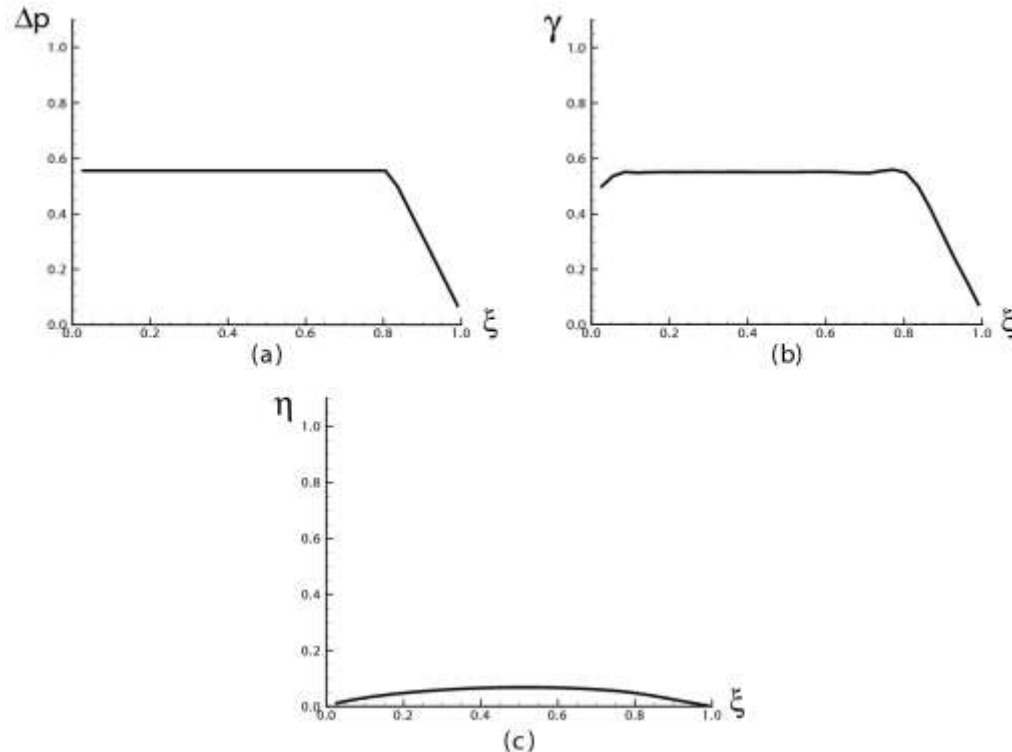


$$\Delta p = -\rho U \gamma$$

$$u^{\pm} = \mp \frac{\gamma}{2}$$



# Pressure and Vorticity Distribution



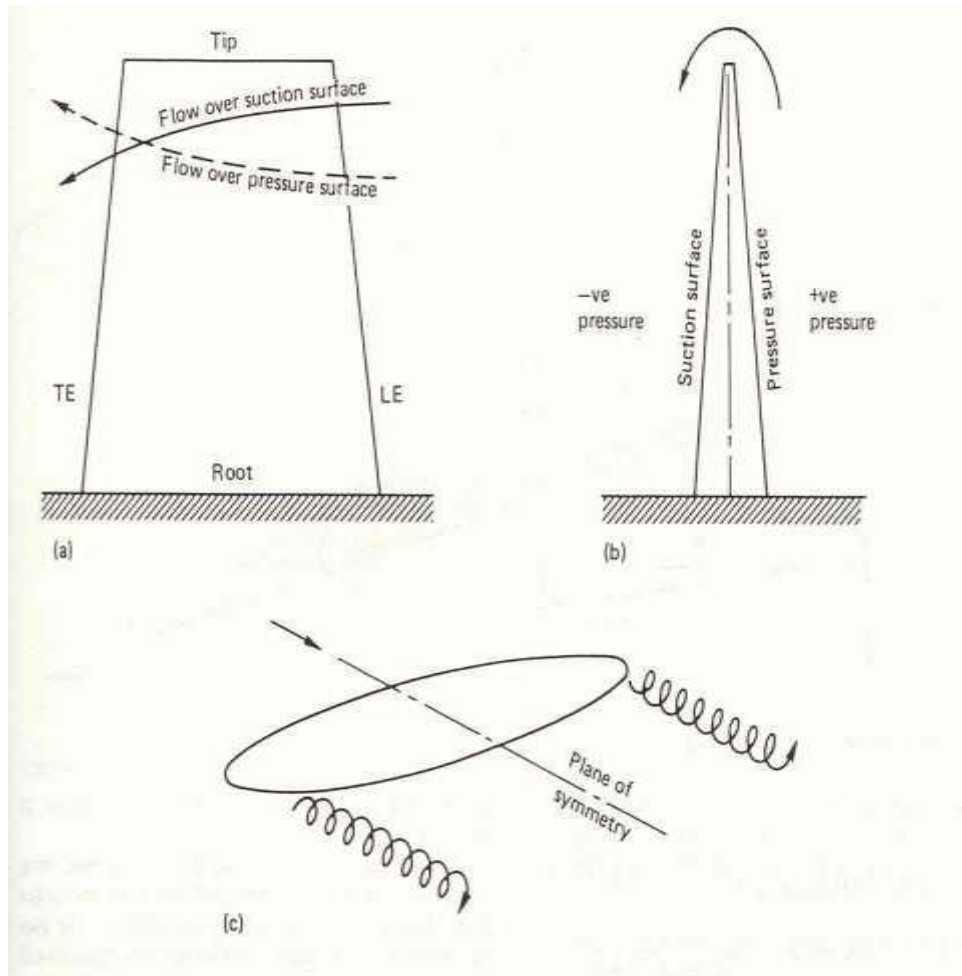
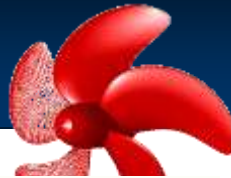
NACA a=0.8 meanline

Data for NACA mean line a=0.8 (modified)

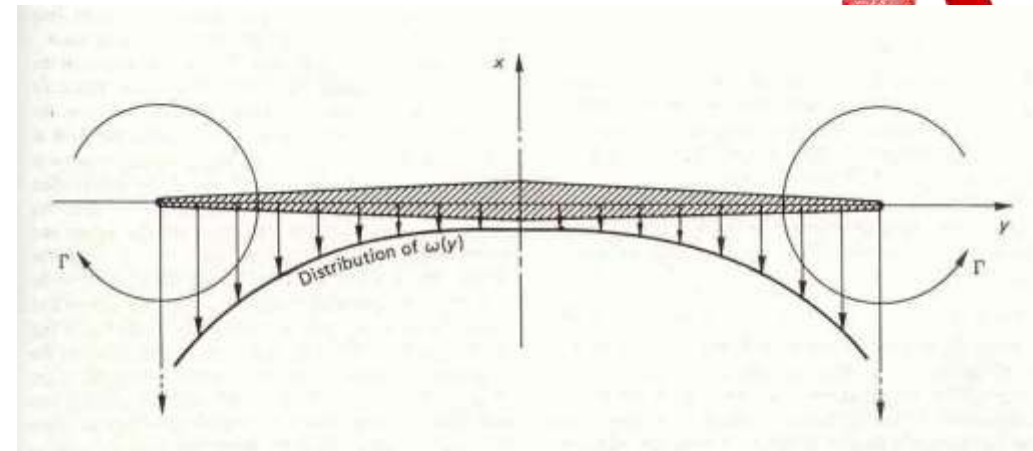
$c_{Li} = 1.0, \alpha_i = 1.40^\circ, c_{m(c/4)} = 0.219$					
$x$ (% c)	$y_c$ (% c)	$-\Delta C_p$	$x$ (% c)	$y_c$ (% c)	$-\Delta C_p$
0	0		40	6.394	1.100
0.5	0.281	1.092	45	6.571	1.100
0.75	0.396	1.092	50	6.651	1.104
1.35	0.603	1.092	55	6.631	1.104
2.5	1.055	1.092	60	6.508	1.104
5.0	1.803	1.092	65	6.274	1.108
7.5	2.432	1.092	70	5.913	1.108
10	2.981	1.092	75	5.401	1.112
15	3.903	1.096	80	4.673	1.112
20	4.651	1.096	85	3.607	0.840
25	5.257	1.096	90	2.452	0.558
30	5.742	1.096	95	1.226	0.368
35	6.120	1.100	100	0	0

Local load coefficient:  $\Delta C_p = \Delta p / 0.5 \rho U^2$

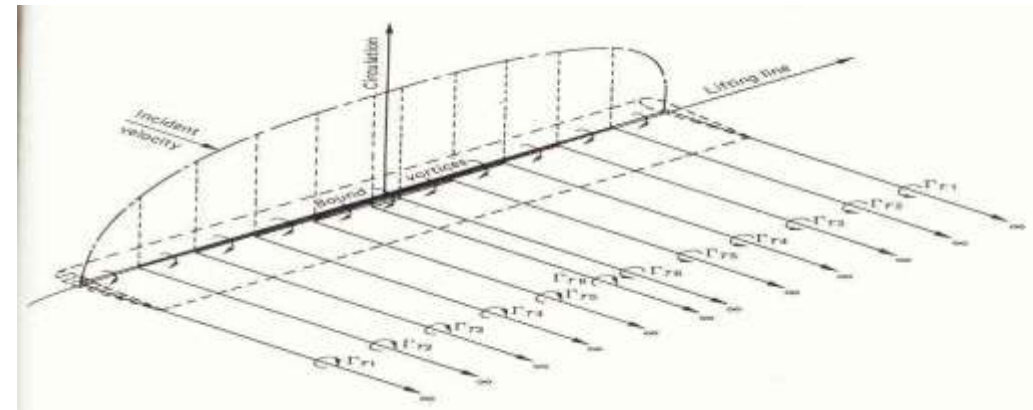
# Finite wing



Flow over a finite aspect ratio wing : (a) plan view of blade; (b) flow at blade tip; (c) schematic view of wing tip vortices

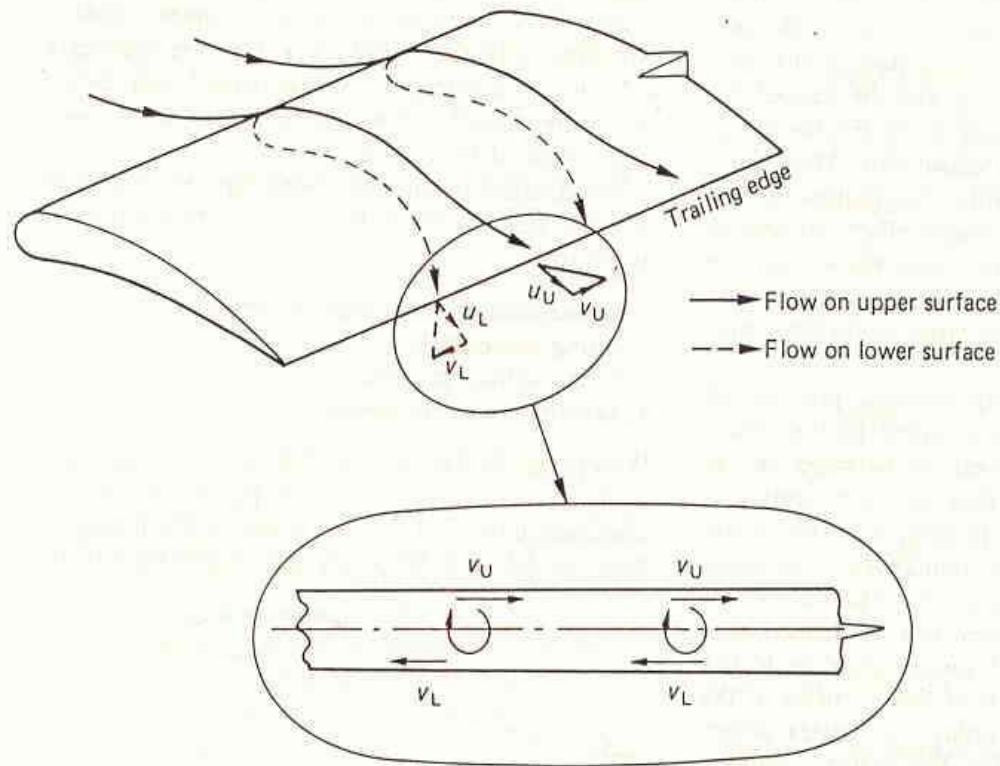


Downwash distribution for a pair of tip vortices on a finite wing

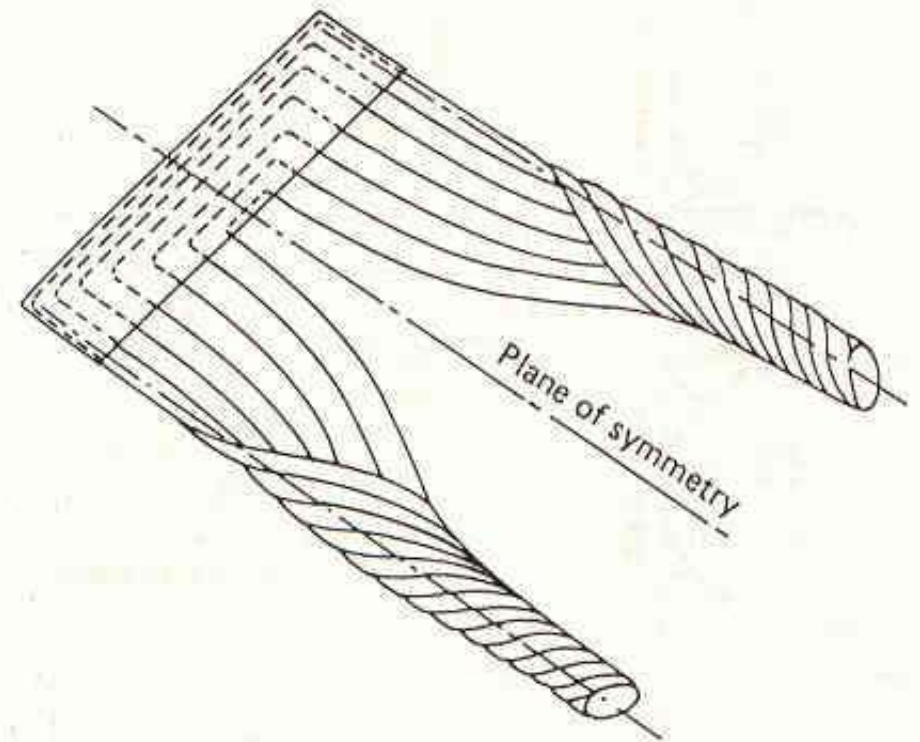


Prandtl's classical lifting line theory

# Finite wing



Formation of trailing vortices



Schematic roll-up of trailing vortices



# Wake vortex distribution



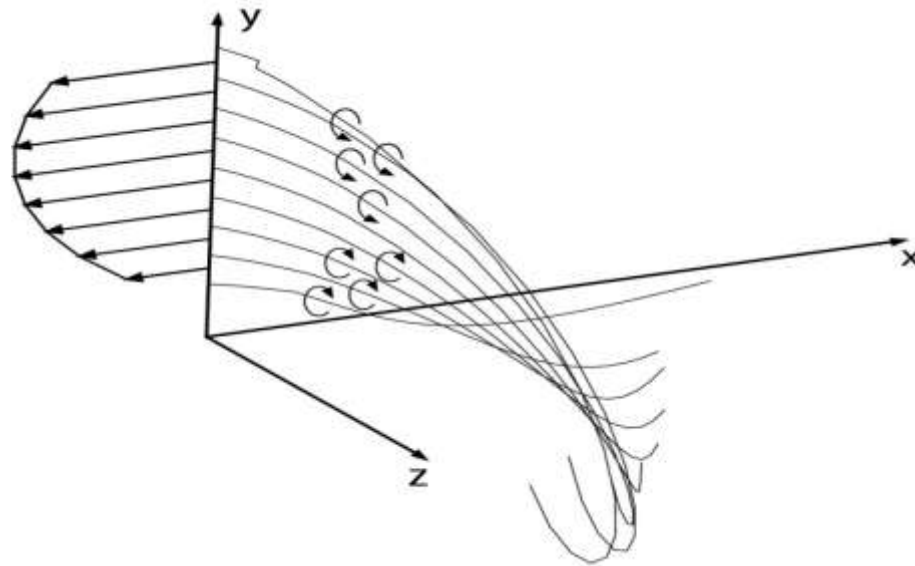
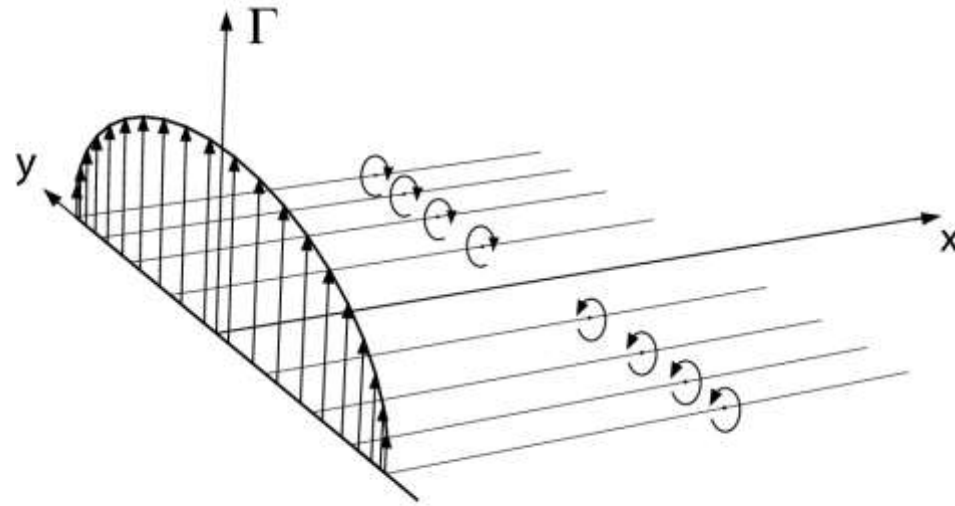
❖ Kelvin's circulation conservation law

❖ Free vortex in wing

$$\gamma_y^t = -\frac{\partial \Gamma}{\partial y}$$

❖ Free vortex in propeller

$$\gamma_y^t = -\frac{\partial \Gamma}{\partial r}$$





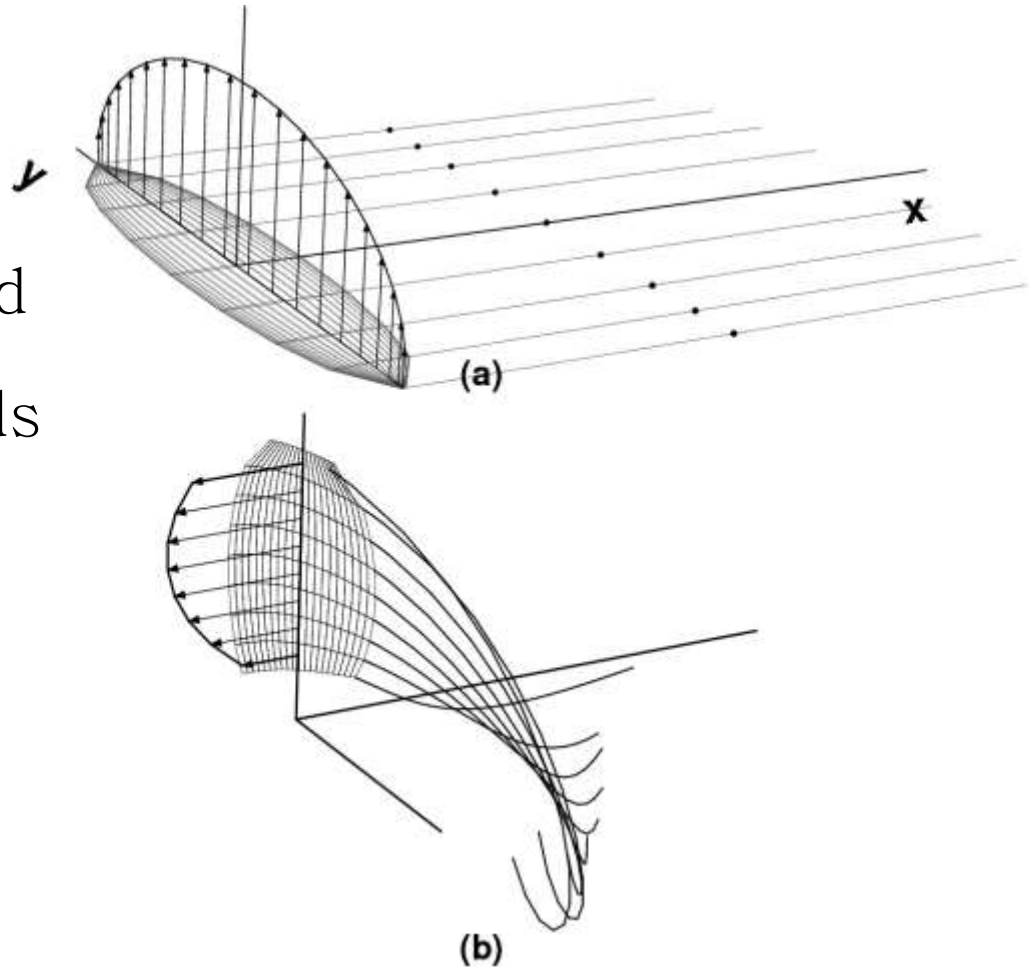
# VLM(보오텍스격자법)에 의한 프로펠러 해석



❖ Vortex Lattice Method

❖ Lifting surface method

→ Lifting line methods

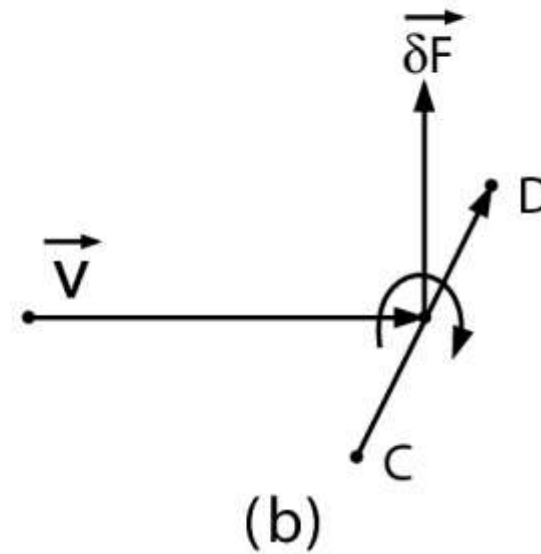
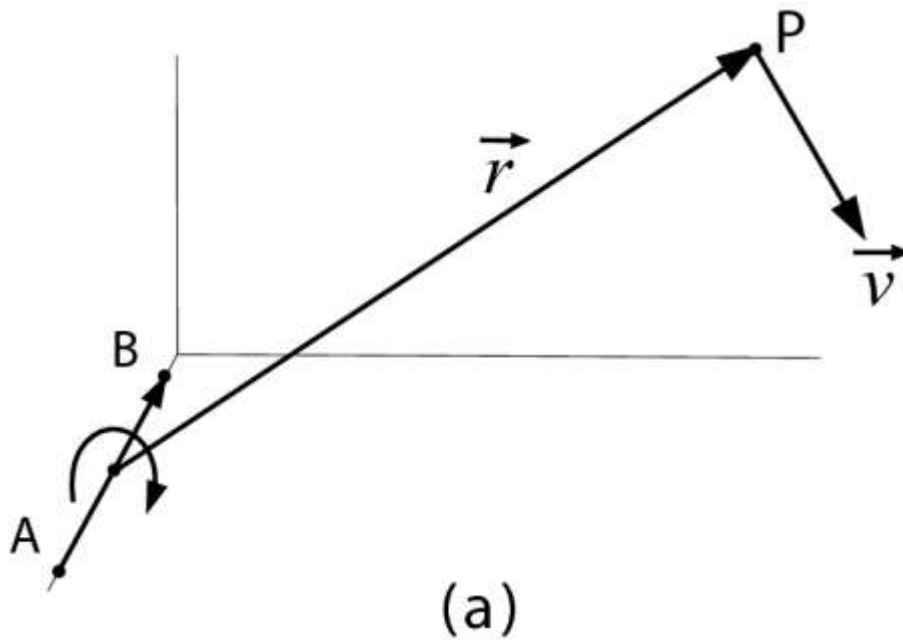


# Induced velocity(Biot-Savart Law)



$$\vec{v} = \frac{\Gamma}{4\pi} \int_A^B \frac{d\vec{s} \times \vec{r}}{|\vec{r}|^3}$$

$$\delta \vec{F} = \rho \vec{V} \times \Gamma \delta \vec{\ell}_C^D$$

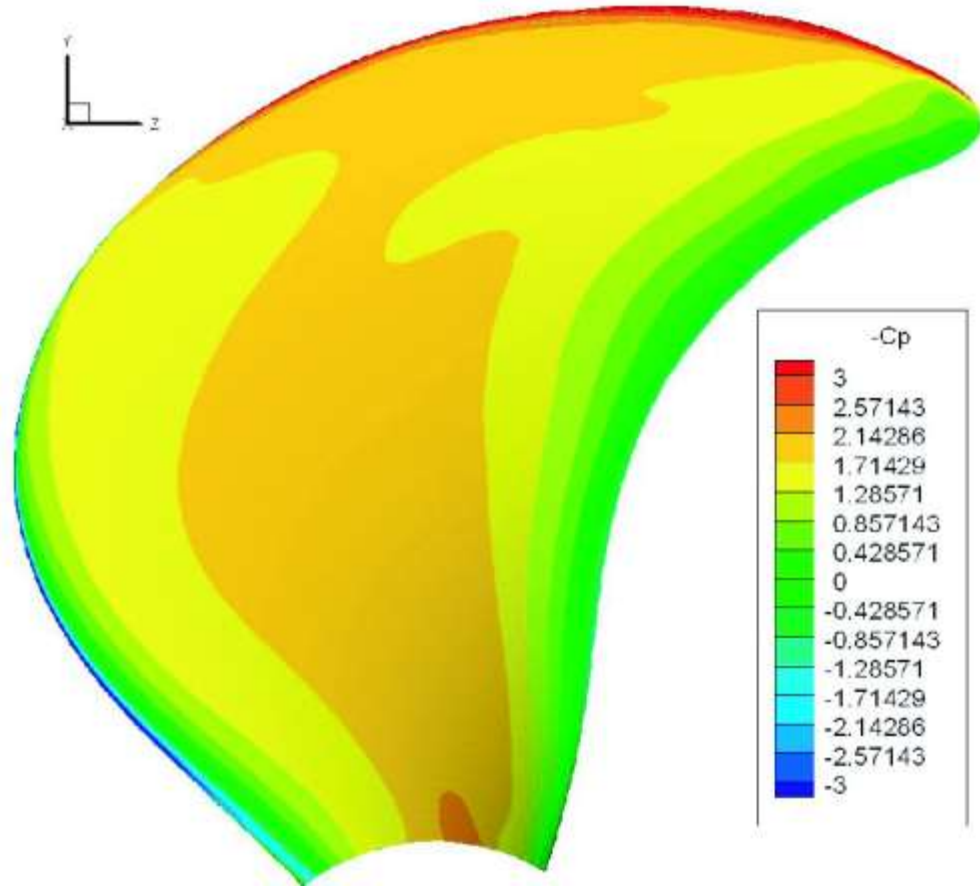


# Example of pressure computation with panel method

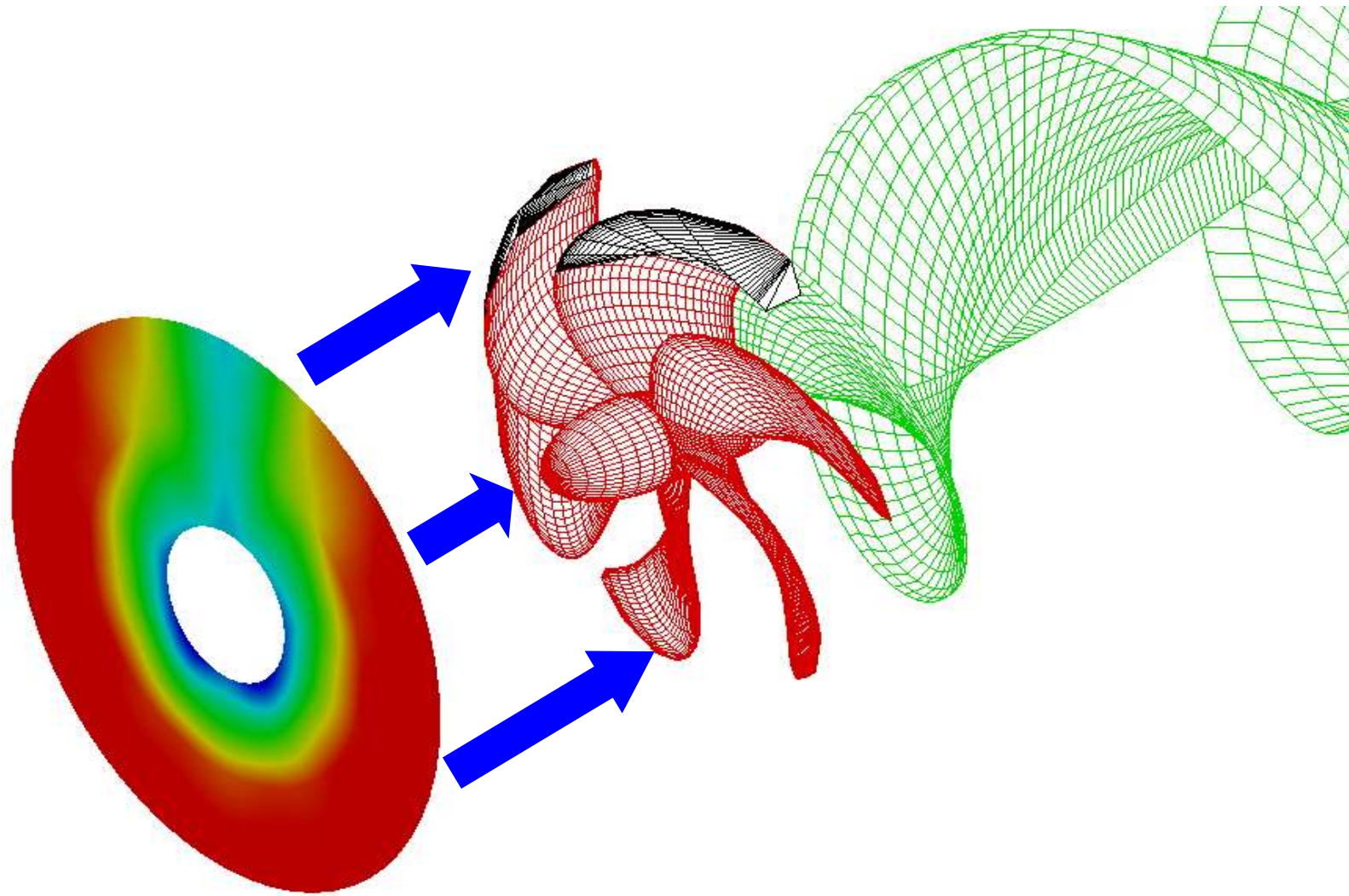


$$\vec{F} = -Z \iint p \hat{n} dS$$

$$T = -\hat{i} \cdot \vec{F} = -F_x$$



# Propeller-Cavity Analysis



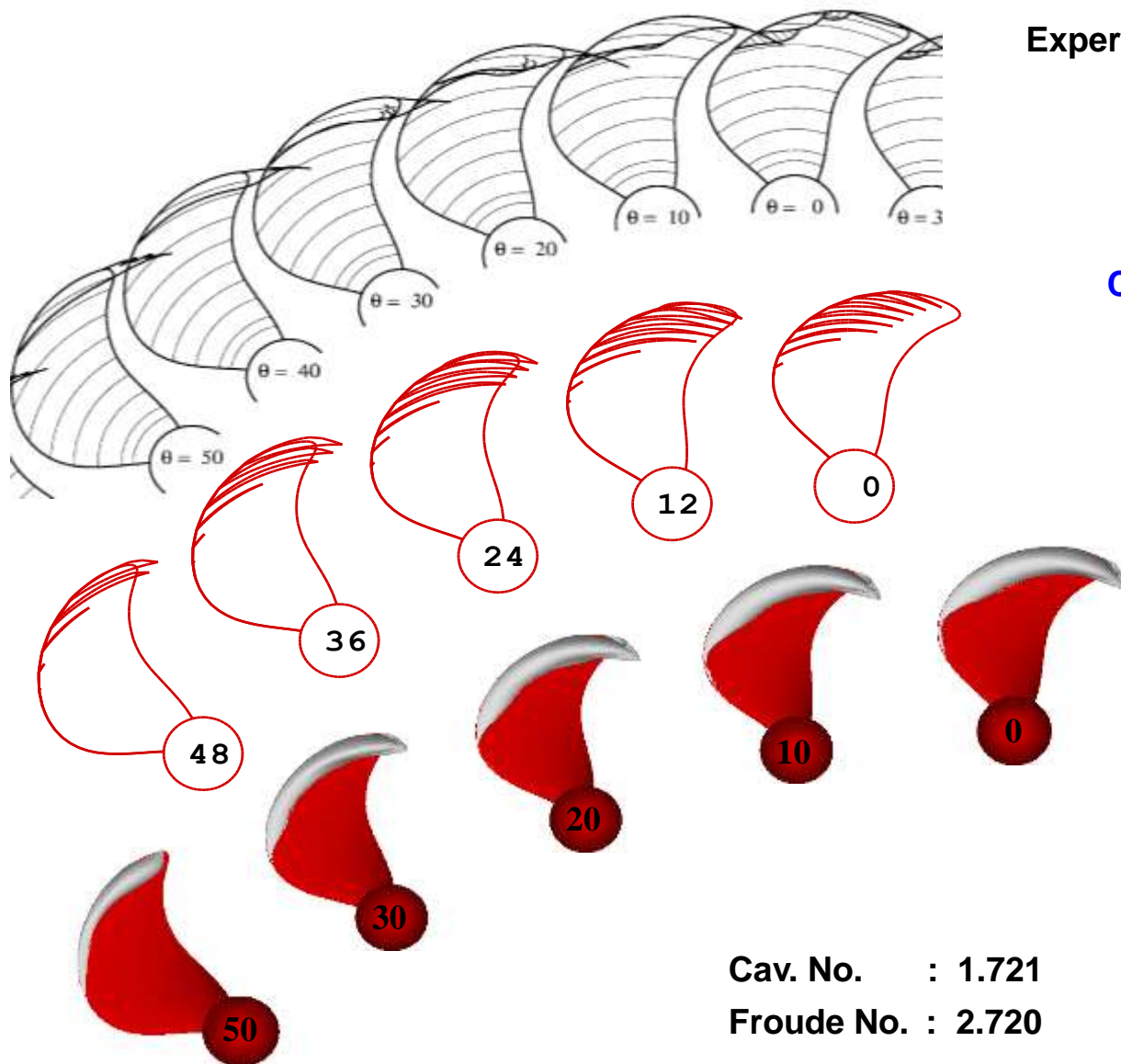
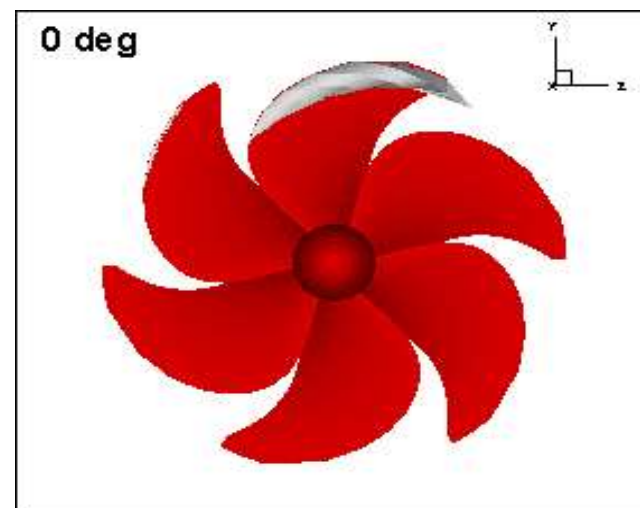
# Cavity Prediction



Experiment

Calculated Cavity Patterns  
by VLM code

Calculated Cavity Patterns  
by PANEL code



Cav. No. : 1.721

Froude No. : 2.720